Algebraic Integer Encoding and Applications in Discrete Cosine Transform

Minyi Fu

Supervisors: Dr. G. A. Jullien
Dr. M. Ahmadi

Department of Electrical and Computer Engineering
University of Windsor

Feb. 3rd, 2004
OUTLINE

• Algebraic Integer DCT Encoding
• DCT IP Core Design and Fabrication
• Simulation Results and Chip Testing
• Conclusion
### DCT

**DCT:**

1-D DCT: \[ F(k) = \sum_{n=0}^{N-1} x(n) \cdot \cos\left(\frac{(2n+1)k}{2N}\pi\right) \quad 1 \leq k \leq N - 1; \]

2-D DCT: \[ F(k, l) = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} x(m, n) \cdot \cos\left(\frac{(2n+1)k}{2N}\pi\right) \cos\left(\frac{(2m+1)l}{2N}\pi\right); \]

\[ 1 \leq k \leq N - 1 \quad 1 \leq l \leq N - 1 \]

**Properties and Applications:**

- DCT has energy packing capabilities and also approaches the statistically optimal transform in de-correlating a signal governed by Markov Process.
- DCT is orthogonal and separable, it leads to the reduction of spatial redundancy for the input signal and has found wide applications in speech and image processing.
- The 2-Dimensional DCT, over a small block of pixels, has been widely used as a frequency analysis and compression algorithm in image processing standard like MPEG-2.
Algebraic Integer DCT Encoding

\[ Z_1 = 2 \cos(1 \cdot \pi / 16) \]

\[ f(Z_1) = \sum_{i=0}^{7} a_i Z_1^i \]

\[ z_1 = 2 \cos(\pi / 16) \quad z_2 = 2 \cos(4\pi / 16) \]

\[ f(z_1, z_2) = \sum_{i=0}^{3} \sum_{j=0}^{1} a_{ij} z_1^i z_2^j \]

**Table I:** 1D Algebraic Integer Encoding for 8 Point DCT

<table>
<thead>
<tr>
<th></th>
<th>a_0</th>
<th>a_1</th>
<th>a_2</th>
<th>a_3</th>
<th>a_4</th>
<th>a_5</th>
<th>a_6</th>
<th>a_7</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cos(0 \cdot \pi / 16)</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 cos(1 \cdot \pi / 16)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 cos(2 \cdot \pi / 16)</td>
<td>-2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 cos(3 \cdot \pi / 16)</td>
<td>0</td>
<td>-3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 cos(4 \cdot \pi / 16)</td>
<td>2</td>
<td>0</td>
<td>-4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 cos(5 \cdot \pi / 16)</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>-5</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 cos(6 \cdot \pi / 16)</td>
<td>-2</td>
<td>0</td>
<td>9</td>
<td>0</td>
<td>-6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 cos(7 \cdot \pi / 16)</td>
<td>0</td>
<td>-7</td>
<td>0</td>
<td>14</td>
<td>0</td>
<td>-7</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table II:** 2D Algebraic Integer Encoding for 8 Point DCT

\[
\begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Algebraic Integer Encoding and Applications in Discrete Cosine Transform
Exploiting Redundancy – Zero Pattern

\[ F(k) = \sum_{n=0}^{N-1} x(n) \cdot \cos\left( \frac{(2n+1)k}{2N} \pi \right) \]

\[ F(2k') = \sum_{n=0}^{N-1} x(n) \cdot \cos\left( \frac{(2n+1)2k'}{2N} \pi \right); \quad F(2k'+1) = \sum_{n=0}^{N-1} x(n) \cdot \cos\left( \frac{(2n+1)(2k'+1)}{2N} \pi \right); \]

\[ F(0,2,4,6) \quad F(1,3,5,7) \]

\[ \{ \cos\left( \frac{0\pi}{16} \right), \cos\left( \frac{2\pi}{16} \right), \cos\left( \frac{4\pi}{16} \right), \cos\left( \frac{6\pi}{16} \right) \} \quad \{ \cos\left( \frac{1\pi}{16} \right), \cos\left( \frac{3\pi}{16} \right), \cos\left( \frac{5\pi}{16} \right), \cos\left( \frac{7\pi}{16} \right) \} \]
Exploiting Redundancy – Zero Pattern

2D implementation:
15 layers of algebraic integer representation

1D implementation:
8 layers of algebraic integer representation

Zero Pattern:
4 layers of algebraic integer representation
Algebraic Integer Encoding and Applications in Discrete Cosine Transform
• Function       two-dimensional 8x8 DCT
• Inputs / Outputs 9 bit signed(pixel)/12 bit signed (DCT)
• Internal Word-length 10-13 (algebraic integer), 16 (binary)
• Accuracy       IEEE Standard 1180-1990
• Technology     TSMC CMOS 0.18µm
• Core Size       1.8mm × 1.2mm
• Power Dissipation 7.5mW @ 75MHz/1.2V
• Throughput      75M pixel/second
• Latency         80 clock cycles

Algebraic Integer 8x8 DCT Chip Micrograph and Highlights
Simulation Results - numerical characteristics

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>mppe</td>
<td>&lt;=1</td>
<td>&lt;=1</td>
<td>&lt;=1</td>
<td>&lt;=1</td>
</tr>
<tr>
<td>mpmse/mpme</td>
<td>0.055</td>
<td>0.056</td>
<td>0</td>
<td>&lt;=0.06</td>
</tr>
<tr>
<td>ome/omse</td>
<td>0.00072</td>
<td>0.00084</td>
<td>0</td>
<td>&lt;=0.0015</td>
</tr>
<tr>
<td>zero_test</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Simulation Results According to IEEE Standard 1180-1990
Using Algebraic Integer Representations
# Simulation Results – Power Estimation

## Power Consumption for Processing Input Image Blocks of 128x128

<table>
<thead>
<tr>
<th>Image \ Design</th>
<th>ICFWRDCT</th>
<th>DCT_Inc_Compile</th>
<th>clock_gating1</th>
<th>clock_gating2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lena</td>
<td>8.536/4.802</td>
<td>6.139/3.453</td>
<td>5.034/2.832</td>
<td>4.584/2.579</td>
</tr>
<tr>
<td>Bird</td>
<td>7.570/4.258</td>
<td>5.442/3.061</td>
<td>4.377/2.462</td>
<td>4.084/2.297</td>
</tr>
</tbody>
</table>

Global Operating Voltage = 1.6/1.2 V  
Operating Speed: 75 MHz  
Power Unit: mW
CMC DUT Testing Board on the CMC TH1000 Test Head

Testing Environment

- CMC TH1000 Test Head
- HP 9000/745i workstation with HP-UX A09.01 Operating System
- HP 75000D20, VXI Digital Test System
- HP 6621A DC Power Supplies
- Tektronix 11402 Digital Oscilloscope
Simulation Results and Chip Testing

HP Veetest Digital Testing Software Environment
Simulation Results and Chip Testing

IMS Digital Testing System Environment
Simulation Results and Chip Testing

- Functional: Works.
- Test Frequency: 50MHz.
- Power Consumption: 1.8V*0.0063mA = 11.34mW @ 50MHz

scaling: 1.2V*0.0042mA = 5.04mW @ 50MHz
7.56mW @ 75MHz

<table>
<thead>
<tr>
<th>Design</th>
<th>Core Size / Technology</th>
<th>Scaled Power Consumption (mJ/Mpixels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xanthopoulos [5]</td>
<td>14.5mm² / 0.6μm CMOS</td>
<td>0.313</td>
</tr>
<tr>
<td>Chang et al. [6]</td>
<td>7.85 × 6.45 mm² / 0.6μm CMOS</td>
<td>1.38</td>
</tr>
<tr>
<td>August et al. [7]</td>
<td>0.35μm CMOS</td>
<td>0.156</td>
</tr>
<tr>
<td>Masera et al. [8]</td>
<td>Xilinx XCV100E</td>
<td>0.527</td>
</tr>
<tr>
<td>Proposed Alg_int DCT</td>
<td>1.8mm × 1.2mm / 0.18μm CMOS</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Testing Results and Power Consumption Comparisons
Conclusion

- The error-free 2D algebraic integer encoding scheme for DCT basis function provide an alternative for DCT computing

- The multiplier-less high-precision feature of the algebraic integer encoding combined with selected suitable DCT algorithm enable an efficient implementation of the 8 x 8 DCT IP core
Publications


